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1990 J. Phys.: Condens. Matter 2 SA469

(http://iopscience.iop.org/0953-8984/2/S/075)

J. Phys.: Condens. Matter 2 (1990) SA469–SA475. Printed in the UK

Large-scale structures in turbulent multiphase flows

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Received 9 July 1990

Abstract. A generation of large-scale helical vortices resulting from the instability of small-scale helical turbulence with respect to two-scale disturbances is considered. To investigate such an instability we consider an incompressible liquid containing rigid particles. An equation describing the evolution of mean disturbances is derived and the instability increment is obtained.

1. Introduction

Magnetic disturbances are known to be amplified by helical turbulence [1]. The possibility of amplification of large-scale hydrodynamic fields by small-scale helical turbulence is discussed by several researchers [2–4]. As in the magnetic case Moiseev *et al* [2] call this phenomenon a hydrodynamic α effect, whose existence demands that additional factors be taken into consideration, for instance, convection or compressibility. The important difference between hydrodynamic and magnetic theories is that the latter describe the evolution of magnetic field on the background of a given hydrodynamic flow, whereas in hydrodynamics such a situation is more complex. The hydrodynamic problem is self-consistent and non-linear. Several workers [2–4] overcame that difficulty by introducing an external force which creates a turbulent flow with the necessary properties. A generation of large-scale helical vortices resulting from the instability of small-scale helical turbulence with respect to two-scale disturbance is considered. In order to investigate such an instability we consider an incompressible liquid containing rigid particles.

We restrict ourselves to the case when (i) the characteristic rigid particle dimension significantly exceeds all the kinetic scales and (ii) the disturbance scales are sufficiently large. We can thus consider rigid particles to form a continuous medium, in which case we can use the equations of two-phase hydrodynamics [5, 6]. Later, while deriving the averaged equations, we shall often make use of the fact that the relative volume occupied by all particles is small enough in order for the collisions between particles to be neglected.

2. Basic equations

To describe the motion of viscous incompressible fluid we use the following set of equations:

$$\partial n/\partial t + \operatorname{div} n\mathbb{V} = 0 \tag{1}$$

$$\operatorname{div} \mathbb{V} = -\tfrac{4}{3}\pi a^3 \operatorname{div} n(\mathbb{V}_s - \mathbb{V}) \tag{2}$$

$$\partial \mathbb{V}_s/\partial t + \beta(\mathbb{V}_s - \mathbb{V}) + (\mathbb{V}_s\nabla)\mathbb{V}_s + (1/\rho)\nabla P = 0 \tag{3}$$

$$\partial \mathbb{V}/\partial t + \gamma n(\mathbb{V} - \mathbb{V}_s) + (\mathbb{V}\nabla)\mathbb{V} + (1/\rho)\nabla P = \nu\nabla^2\mathbb{V} \tag{4}$$

where $\mathbb{V}$ and $\mathbb{V}_s$ are the hydrodynamic velocities of liquid and solid phases, respectively; ρ and ρ_s are their densities; n is the concentration of rigid particles with radius a; ν the kinematic viscosity and P the liquid pressure.

The rigid phase particles are assumed not to interact with each other. The coefficients

$$\gamma = \tfrac{4}{3}\pi a^3\beta\, \rho_s/\rho \qquad \beta = \tfrac{9}{2}(\nu/a^2)\rho/\rho_s$$

characterize Stokes' friction between the phases.

Equation (1) is the continuity equation for the rigid particles; equation (2) describes the fluid displacement produced by rigid particles, which results in the divergence of an incompressible liquid. The rest of the equations of the set describe the momentum conservation law for each of the phases.

In the set of equations (1)–(4) the Stokes friction is taken into consideration only, while the Basse force and the effects due to joint mass force are neglected. This assumption is true when the characteristic timescales of the processes studied exceed the time of establishing the quasi-steady (Stokes) velocity field in a carrying phase around the particles, and less than the time of establishing the phase velocity equilibrium [5].

The two-scale approach used is based on the assumption that the perturbed mean field of velocities has characteristic scales L and T and changes slowly at the scales l_0 and t_0 of a turbulent field. Our main interest is concerned with the evolution of the ensemble averaged fluctuation field during the time and space scales which are much greater than the energy-containing vortices scales of primary turbulence.

The variables of the basic set of equations are in the following form:

$$\mathbb{V} = \langle\mathbb{V}\rangle + \mathbb{V}' \qquad \mathbb{V}_s = \langle\mathbb{V}_s\rangle + \mathbb{V}_s' \qquad n = \langle n\rangle + n' \qquad P = \langle P\rangle + P'$$

where averaging over turbulent fluctuation ensemble is designated by $\langle\ \rangle$ and the turbulent component is denoted by a prime.

The equations for the mean flows

$$\frac{\partial\langle n\rangle}{\partial t} + \operatorname{div}\langle n\rangle\langle\mathbb{V}_s\rangle + \operatorname{div}\langle n'\mathbb{V}_s\rangle = 0 \tag{5}$$

$$\operatorname{div}\langle\mathbb{V}\rangle = \tfrac{4}{3}\pi a^3\, \partial\langle n\rangle/\partial t + \tfrac{4}{3}\pi a^3 \operatorname{div}\langle\mathbb{V}\rangle\langle n\rangle + \tfrac{4}{3}\pi a^3 \operatorname{div}\langle\mathbb{V}'n'\rangle \tag{6}$$

$$\partial\langle\mathbb{V}_s\rangle/\partial t + \beta(\langle\mathbb{V}_s\rangle - \langle\mathbb{V}\rangle) + (\langle\mathbb{V}_s\rangle\nabla)\langle\mathbb{V}_s\rangle + \langle(\mathbb{V}_s'\nabla)\mathbb{V}_s'\rangle + (1/\rho_s)\nabla\langle P\rangle = 0 \tag{7}$$

$$\begin{aligned}\partial\langle\mathbb{V}\rangle/\partial t &+ \gamma\langle n\rangle(\langle\mathbb{V}\rangle - \langle\mathbb{V}_s\rangle) + \gamma\langle n'(\mathbb{V}' - \mathbb{V}_s')\rangle + (\langle\mathbb{V}\rangle\nabla)\langle\mathbb{V}\rangle\\ &+ \langle(\mathbb{V}'\nabla)\mathbb{V}'\rangle + (1/\rho)\nabla\langle P\rangle = \nu\nabla^2\langle\nu\rangle\end{aligned} \tag{8}$$

contain the terms with the unknown Reynolds stresses and are supplemented by the corresponding equations for the pulsing components.

$$\partial n'/\partial t + \operatorname{div}(n'\langle \mathbb{V}_s\rangle + \langle n\rangle \mathbb{V}_s) + \operatorname{div}(n'\mathbb{V}_s' - \langle n'\mathbb{V}_s'\rangle) = 0 \tag{9}$$

$$\operatorname{div}\mathbb{V}' = \tfrac{4}{3}\pi a^3\, \partial n'/\partial t + \tfrac{4}{3}\pi a^3 \operatorname{div}(n'\mathbb{V}' - \langle n'\mathbb{V}'\rangle) + \tfrac{4}{3}\pi a^3 \operatorname{div}(n'\langle \mathbb{V}\rangle + \langle n\rangle \mathbb{V}) \tag{10}$$

$$\begin{aligned}\partial \mathbb{V}_s'/\partial t &+ \beta(\mathbb{V}_s' - \mathbb{V}') + ((\langle \mathbb{V}_s\rangle\nabla)\mathbb{V}_s' + (\mathbb{V}_s'\nabla)\langle \mathbb{V}_s\rangle \\ &+ ((\mathbb{V}_s'\nabla)\mathbb{V}_s - \langle(\mathbb{V}_s'\nabla)\mathbb{V}_s\rangle) + (1/\rho_s)\nabla P' = 0\end{aligned} \tag{11}$$

$$\begin{aligned}&\partial \mathbb{V}'/\partial t + \gamma\langle n\rangle(\mathbb{V}' - \mathbb{V}_s') + \gamma n'(\langle \mathbb{V}_s\rangle - \langle \mathbb{V}\rangle) + ((\langle \mathbb{V}\rangle\nabla)\mathbb{V}' + (\mathbb{V}'\nabla)\langle \mathbb{V}\rangle \\ &+ ((\mathbb{V}'\nabla)\mathbb{V} - \langle(\mathbb{V}'\nabla)\mathbb{V}\rangle) + [\gamma n'(\mathbb{V}' - \mathbb{V}_s') - \gamma\langle n'(\mathbb{V}' - \mathbb{V}_s')\rangle] \\ &\qquad + (1/\rho)\nabla P' = \nu\nabla^2\mathbb{V}.\end{aligned} \tag{12}$$

Equations (9)–(12) will be used later to derive the closed form equations describing mean motions.

In the following the ergodic condition is supposed true, which allows us to replace the ensemble average by the local average over space and so to put the problem of disturbance evolution at the background of initial state in equations (5)–(12).

3. The derivation of the averaged equations for disturbances

Consider the initial state which is characterized by the mean rigid particle concentration n_0 and small-scale fluid velocities $\mathbb{V}'^0$ and of particles $\mathbb{V}_s'^0$. The following assumptions concerning the hydrodynamic fields are made.

(i) The liquid phase turbulence is supposed to be homogeneous, isotropic and helical with a correlation tensor:

$$\langle \mathbb{V}_i'^0(x,t)\mathbb{V}_j'^0(x+\xi, t+\tau)\rangle = A(\xi,\tau)\varepsilon_{ijk}\xi_k \tag{13}$$

where ε_{ijk} is the Levy–Chivity tensor. The effect of the symmetrical part of the correlation tensor was studied in [7], where it was shown that it resulted in additional viscosity carried in by turbulence and manifested itself in changing the factor ν. These properties of turbulence are due to the external source in our problem.

(ii) Undoubtedly such a helical turbulent velocity field results in hydrodynamic motions of rigid phase. However, our assumptions about the Stokes friction between phases and neglect of interactions between particles provide us with a possibility of neglecting the correlation between the liquid turbulent field and the hydrodynamic field of rigid particles. For the same reason we neglect the correlation between rigid particles' hydrodynamic velocities in liquid. This corresponds to the melting of rigid particles in the liquid.

Within the framework of the two-scale model proposed, let us consider the evolution of disturbances at the background of the initial state described, which is reduced to the following equations:

$$\langle n\rangle = n^0 \qquad \langle \mathbb{V}'\rangle = \langle \mathbb{V}_s'\rangle = 0 \qquad \operatorname{div}\mathbb{V}'^0 = \operatorname{div}\mathbb{V}_s'^0 = 0$$

$$\partial \mathbb{V}_s'^0/\partial t + \beta(\mathbb{V}_s'^0 - \mathbb{V}'^0) + (\mathbb{V}_s'^0\nabla)\mathbb{V}_s'^0 + (1/\rho_s)\nabla P'^0 = 0$$

$$\partial \mathbb{V}'^0/\partial t + \gamma n^0(\mathbb{V}'^0 - \mathbb{V}_s'^0) + (\mathbb{V}'^0\nabla)\mathbb{V}'^0 + (1/\rho)\nabla P'^0 = \nu\nabla^2\mathbb{V}'^0 + F.$$

Later the closed form equations describing the behaviour of mean disturbances will be obtained by means of linearization of the basic equations.

The perturbed small-scale motions are described as follows:

$$\mathbb{V}' = \mathbb{V}^0 + \mathbb{V}^1 \qquad \mathbb{V}_s' = \mathbb{V}_s^0 + \mathbb{V}_s^1 \qquad n' = n^1 \qquad P' = P^0 + P^1$$

where the hatches for the perturbed components are omitted.

Omitting the non-linear terms in equations (5)–(8) we obtain

$$\partial\langle n\rangle/\partial t + \operatorname{div} n^0\langle\mathbb{V}_s\rangle + \operatorname{div}\mathbb{N}_s = 0 \tag{14}$$

$$\operatorname{div}\langle\mathbb{V}\rangle = \tfrac{4}{3}\pi a^3\,\partial\langle n\rangle/\partial t + \tfrac{4}{3}\pi a^3 \operatorname{div} n_0\langle\mathbb{V}\rangle + \tfrac{4}{3}\pi a^3 \operatorname{div}\mathbb{N} \tag{15}$$

$$\partial\langle\mathbb{V}_s/\partial t + \beta(\langle\mathbb{V}_s\rangle - \langle\mathbb{V}\rangle) + \mathbb{Q}_s + (1/\rho_s)\nabla\langle P\rangle = 0 \tag{16}$$

$$\partial\langle\mathbb{V}\rangle/\partial t + \gamma\langle n\rangle(\langle\mathbb{V}\rangle - \langle\mathbb{V}_s\rangle) + \gamma(\mathbb{N} - \mathbb{N}_s) + \mathbb{Q} + (1/\rho)\nabla\langle P\rangle = \nu\nabla^2\langle\mathbb{V}\rangle \tag{17}$$

where $\langle n\rangle$ is the mean perturbation of the concentration of particles and $\langle v\rangle$ and $\langle\mathbb{V}_S\rangle$ are the perturbation of hydrodynamic velocities.

The above set of equations contains the unknown terms

$$\mathbb{Q} = \langle(\mathbb{V}^1\nabla)\mathbb{V}^0 - (\mathbb{V}^0\nabla)\mathbb{V}^1\rangle \qquad \mathbb{Q}_S = \langle(\mathbb{V}_s^1\nabla)\mathbb{V}_s^0 - (\mathbb{V}_s^0\nabla)\mathbb{V}_s^1\rangle$$

$$\mathbb{N} = \langle n^1\mathbb{V}^0\rangle \qquad \mathbb{N}_s = \langle n^1\mathbb{V}_s^0\rangle.$$

While calculating them we consider symmetry over the indexes part as tending to zero for the helical turbulence:

$$Q_i = \langle(\partial/\partial x_k)(\mathbb{V}_k^1\mathbb{V}_i^0 + \mathbb{V}_i^1\mathbb{V}_k^0)\rangle - \langle\mathbb{V}_i^0(\partial/\partial x_k)\mathbb{V}_k^1\rangle$$

$$Q_{s_i} = \langle(\partial/\partial x_k)(\mathbb{V}_{s_k}^1\mathbb{V}_{s_i}^0 + \mathbb{V}_{s_i}^1\mathbb{V}_{s_k}^0)\rangle - \langle\mathbb{V}_{s_i}^0(\partial/\partial x_k)\mathbb{V}_{s_k}^1\rangle.$$

The methods introduced in [7] for the calculation of the non-symmetrical parts of the correlators are used; for this aim we present equations (9)–(12) in the following form:

$$\partial n^1/\partial t + \operatorname{div}\langle n\rangle\mathbb{V}_s^0 + n^0 \operatorname{div}\mathbb{V}_s^1 = 0 \tag{18}$$

$$\operatorname{div}\mathbb{V}^1 = \tfrac{4}{3}\pi a^3\,\partial n^1/\partial t + \tfrac{4}{3}\pi a^3 \operatorname{div}(\mathbb{V}^0\nabla)\langle n\rangle \tag{19}$$

$$\partial\mathbb{V}_s^1/\partial t + \beta(\mathbb{V}_s^1 - \mathbb{V}^1) + (\langle\mathbb{V}_s\rangle\nabla)\mathbb{V}_s^0 + (\mathbb{V}_s^0\nabla)\langle\mathbb{V}_s\rangle + (1/\rho_s)\nabla P^1 = 0 \tag{20}$$

$$\partial\mathbb{V}^1/\partial t + \gamma n^0(\mathbb{V}^1 - \mathbb{V}_s^1) + \gamma\langle n\rangle(\mathbb{V}^0 - \mathbb{V}_s^0) + (\langle\mathbb{V}\rangle\nabla)\mathbb{V}^0 + (\mathbb{V}^0\nabla)\langle\mathbb{V}\rangle + (1/\rho)\nabla P^1 = \nu\nabla^2\mathbb{V}^1. \tag{21}$$

The terms which, in the following calculations, result in correlation moments of greater than second order are omitted in the system of equations (14)–(17). This allows us to get a closed form of the linearized system for the mean motion and corresponds to the case when the Reynolds number in the initial turbulent flow is $Re_0 \ll 1$.

The assumption concerning the small value of the Reynolds number is related to the turbulence realization parameters and it should not be confused with the Reynolds number of turbulence arising from the laminar flow. As our interest lies in the evolution of small-scale developed turbulence resulting from the initial laminar flow instability, our assumption does not make the physical situation worse. The disturbances appearing for sufficiently large Reynolds numbers which are due to the cut-off procedure should be taken into account. The following expressions for the searched correlators are derived from equations (18)–(21) after simple but lengthy calculations (see appendix)

$$\mathbb{N} = \mathbb{N}_s = \mathbb{Q}_s = 0 \tag{22}$$

$$\mathbb{Q} = \alpha \operatorname{rot}\langle \mathbb{V} \rangle$$

$$\alpha = \tfrac{4}{3}\pi a^3 n^0 \int \left(1 - \frac{\beta \rho_s}{i\omega\rho}\right) \frac{k^4 A(k,\omega)\, \partial k\, \partial\omega}{\{\frac{4}{3}\pi a^3 n^0 [\nu k^2 - (\rho_s/\rho)\beta] + \gamma n^0 (i\omega + \beta)\}}.$$

As a result the following set of closed form equations for mean disturbances is obtained:

$$\partial\langle n\rangle/\partial t + \operatorname{div} n^0 \langle \mathbb{V}_s\rangle = 0 \tag{23}$$

$$\operatorname{div}\langle \mathbb{V}\rangle = \tfrac{4}{3}\pi a^3\, \partial\langle n\rangle/\partial t + \tfrac{4}{3}\pi a^3 n^0 \langle \mathbb{V}\rangle \tag{24}$$

$$\partial\langle \mathbb{V}_s\rangle/\partial t + \beta(\langle \mathbb{V}_s\rangle - \langle \mathbb{V}\rangle) + (1/\rho_s)\nabla\langle P\rangle = 0 \tag{25}$$

$$\partial\langle \mathbb{V}\rangle/\partial t + \gamma\langle n\rangle(\langle \mathbb{V}\rangle - \langle \mathbb{V}_s\rangle) + \alpha \operatorname{rot}\langle \mathbb{V}\rangle + (1/\rho)\nabla\langle P\rangle = \nu\nabla^2\langle \mathbb{V}\rangle. \tag{26}$$

4. The vortical instability in the liquid with rigid particles

The set of equations describing mean disturbances of helical turbulence has been obtained in the previous section. These equations contain the terms which could result in instability. It can be easily seen that these new terms disappear for potential motions and so these are damped.

Focusing our attention on vortical disturbances, we proceed from equations (23)–(26) to the corresponding equations for vorticity:

$$\partial\Omega_s/\partial t + \beta(\Omega_s - \Omega) = 0 \tag{27}$$

$$\partial\Omega/\partial t + \gamma n^0(\Omega - \Omega_s) + \alpha \operatorname{rot}\Omega = \nu\nabla^2\Omega \tag{28}$$

where $\Omega = \operatorname{rot}\langle \mathbb{V}\rangle$, $\Omega_s = \operatorname{rot}\langle \mathbb{V}_s\rangle$.

The term describing the generation of vortices resulting from the instability is present in these equations. The dispersion equation for small disturbances contains two multipliers with helicity

$$\frac{1}{\beta}(-i\omega)^2 - i\omega\left(\frac{\gamma n^0}{\beta} + \frac{\nu k^2}{\beta} \pm \frac{\alpha}{\beta}k + 1\right) + (\nu k^2 \pm \alpha k) = 0 \tag{29}$$

and has a solution describing instability with an increment:

$$-i\omega = -\tfrac{1}{2}(\nu k^2 + \gamma n^0 + \beta - \alpha k) \pm \tfrac{1}{2}[(\nu k^2 + \gamma n^0 + \beta - \alpha k)^2 - 4(\beta\nu k^2 - \alpha k\beta)]^{1/2}.$$

Then we proceed to the limit of the single-phase medium by tending particle radius to zero. Then the factors α, γ and $1/\beta$ will also tend to zero. In this limiting case we obtain $-i\omega = -\nu k^2$ which corresponds to disturbance damping in an incompressible viscous liquid.

5. Conclusion

The analysis revealed that helical turbulence in an incompressible liquid with rigid particles is unstable to vortical disturbances. The generation terms formally coinciding with those in the theory of hydromagnetic dynamics are contained in the equations derived to vorticity at the scale of mean motions. It should be noted that only helicity is enough for the process of generation in magneto hydrodynamics. In hydrodynamic theory, because of the mentioned differences, it is also necessary to take into account additional factors. In this paper one such additional factor is the presence of rigid particles whose motions provide the existence of divergence at a turbulent scale and thus provide a non-zero value of the Reynolds stresses in the averaged equations.

Thus, when the turbulence appears from the initial laminar flow there could arise sufficient conditions for the reverse energy cascade from a small scale to a bigger one.

The instability discovered could be treated as a secondary one which could result in vortical coherent structures.

Acknowledgment

The authors are thankful to Dr B E Nemtsov for fruitful discussions.

Appendix

The equations which describe $\mathbb{Q}$, $\mathbb{Q}_s$, $\mathbb{N}$ and $\mathbb{N}_s$ are obtained from (18)–(21) following Krause *et al* [7]. Substitution of (18) into (19) yields

$$(\partial/\partial x_k)\mathbb{V}_k^1 = -\tfrac{4}{3}\pi a^3(\mathbb{V}_{s_k}^0 - \mathbb{V}_k^0)(\partial/\partial x_k)\langle n\rangle - \tfrac{4}{3}\pi a^3 n^0(\partial/\partial x_k)\mathbb{V}_{s_k}^1. \tag{A.1}$$

Substituting x by $x + \xi$, we make a coordinate displacement in (A.1). Multiplying (A.1) by $\mathbb{V}_0(x, t)$ and keeping it linear with respect to ξ terms (in the two-scale approach the second-order terms and higher-order terms could be omitted) we obtain

$$\langle n(x + \xi, t + \tau)\rangle = \langle n(x, t)\rangle + \xi_k(\partial/\partial x)\langle n(x, t)\rangle.$$

After averaging, the following equations are obtained:

$$\begin{aligned}\langle \mathbb{V}_i^0(x, t)(\partial/\partial x_k)\mathbb{V}_k^1(x + \xi, t + \tau)\rangle &= -\tfrac{4}{3}\pi a^3(\langle \mathbb{V}_i^0(x, t)\mathbb{V}_{s_k}^0(x + \xi, t + \tau)\rangle \\ &\quad - \langle \mathbb{V}_{s_i}^0(x, t)\mathbb{V}_k^0(x + \xi, t + \tau)\rangle)(\partial/\partial x_k)\langle n(x, t)\rangle \\ &\quad - \tfrac{4}{3}\pi a^3 n\langle \mathbb{V}_i^0(x, t)(\partial/\partial x_k)\mathbb{V}_{s_k}^1(x + \xi, t + \tau)\rangle.\end{aligned} \tag{A.2}$$

The Fourier transformation is made in (A.2)

$$\hat{\mathbb{Q}}_{ik}^0(\mathbb{K}, \omega) = \frac{1}{(2\pi)^2}\iint \mathrm{d}\xi\, \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}(\mathbb{K}\xi - \omega t)}\mathbb{Q}_{ik}^0(\xi, \tau)$$

where

$$\mathbb{Q}_{ik}^0(\xi, \tau) = \langle \mathbb{V}_i^0(x, t)\mathbb{V}_k^0(x + \xi, t + \tau)\rangle \qquad \mathbb{Q}_{ik}^{0s}(\xi, \tau) = \langle \mathbb{V}_i^0(x, t)\mathbb{V}_{s_k}(x + \xi, t + \tau)\rangle.$$

The following expression is derived from obtained equations by integrating over $\mathbb{K}$

and ω, and taking into account that $Q^{0s}_{ik} = 0$, and $\mathbb{Q}^{0}_{ik}(\mathbb{K}, \omega)$ equals $iA(k, \omega)\varepsilon_{ikl}k_l$ and becomes zero during integration over angles (due to isotropic initial turbulence)

$$\langle \mathbb{V}^0_i(x,t)(\partial/\partial x_k)\mathbb{V}^1_k(x+\xi, t+\tau)\rangle = -\tfrac{4}{3}\pi a^3 n^0 \langle \mathbb{V}^0_i(x,t)(\partial/\partial x_k)\mathbb{V}^1_{s_k}(x+\xi, t+\tau)\rangle. \tag{A.3}$$

Using a similar method as for (20) and taking into account that div $\mathbb{V}^0 = 0$ we obtain

$$(\partial/\partial\tau + \beta)g^{0s}_i(x,\xi,t,\tau) - \beta g^{00}_i(x,\xi,t,\tau) = -(1/\rho)\nabla^2_\xi \mathbb{P}_i \tag{A.4}$$

$$\begin{aligned}(\partial/\partial\tau + \nu\nabla^2_\xi + \gamma n_0)g^{00}_i(x,\xi,t,\tau) &= -\gamma n_0 g^{0s}_i(x,\xi,t,\tau)\\ &\quad + 2(\partial\langle\mathbb{V}_p\rangle(x,t)/\partial x_k)(\partial/\partial\xi_p)\mathbb{Q}^{00}_{ik}(\xi,\tau) - (1/\rho)\nabla^2_\xi\mathbb{P}_i(x,\xi,t,\tau)\end{aligned} \tag{A.5}$$

where

$$g^{0s}_i(x,\xi,t,\tau) = \langle\mathbb{V}^0_i(x,t)(\partial/\partial x_k)\mathbb{V}^1_{s_k}(x+\xi,t+\tau)\rangle$$
$$g^{00}_i(x,\xi,t,\tau) = \langle\mathbb{V}^0_i(x,t)(\partial/\partial x_k)\mathbb{V}^1_k(x+\xi,t+\tau)\rangle$$
$$\mathbb{P}_i(x,\xi,t,\tau) = \langle\mathbb{V}^0_{s_i}(x,t)P(x,\xi,t,\tau)\rangle.$$

It follows from (A.3)—(A.5) that

$$V(\nu k^2 + \rho/\rho_s)g^{00}_i + \gamma n^0 g^{00}_i + (\rho_s/\rho)(-i\omega + \beta)g^{00}_i = -2(\partial\mathbb{V}_p/\partial x_k)(\partial/\partial\xi_p)\mathbb{Q}^{00}_{ik}V$$

where $V = \frac{4}{3}\pi n^0 a^3$; the expression for g^{00}_i is obtained in (x, t) representation:

$$g^{00}_i(x,t) = \varphi \text{ rot } \mathbb{V}(x,t) \tag{A.6}$$

$$\varphi = \tfrac{2}{3}V\int k^4 A(k,\omega)\,\partial k\,\partial\omega\left[\tfrac{4}{3}V\left(\nu k^2 + \frac{\rho_s}{\rho}\beta\right) + \gamma n_0 + \frac{\rho_s}{\rho}(i\omega+\beta)\right]^{-1}. \tag{A.7}$$

The analogous calculations for (19) yield

$$g_{ik}(x,\xi,t,\tau) = V(\partial/\partial\tau)\mathbb{N}_i(x,\xi,t,\tau) + V\mathbb{Q}_{ik}(\xi,\tau)(\partial/\partial x_k)\langle n(x,t)\rangle$$

which gives $\mathbb{N}_i(x,0,t,0) = \beta \text{ rot}\langle\mathbb{V}\rangle(x,t)$ where

$$\beta = \tfrac{4}{3}V\frac{1}{2\pi a^3}\,i\int k^4 A(k,\omega)\,\partial k\,\partial\omega\left[\omega\left(\tfrac{4}{3}V(\nu k^2 + \frac{\rho_s}{\beta}\right) + \gamma n_0 + \frac{\rho_s}{\rho}(i\omega+\beta)\right)\right]^{-1}.$$

All other additional terms contained in averaged equations become zero due to the equivalence of $\mathbb{Q}^{0s}(x,\xi,t,\tau)$ and $\mathbb{Q}^{0s}(x,\xi,t,\tau)$ to zero.

References

[1] Vainshtein S I, Zeldovich Y B and Ruzmaikin A A 1980 *Turbulent Dynamo in Astrophysics* (Moscow: Nauka)

[2] Moiseev S S, Sagdeev R Z, Tur A V, Khomenko G A and Yanovsky V V 1983 *Sov. Phys.-JETP* **85** 1979

[3] Moiseev S S, Oganyan K R, Rutkevich P B, Tur A V, Khomenko G A and Yanovsky V V 1990 *Integrability and Kinetic Equations for Solitons* (Kiev: Naukova Dumka)

[4] Gvaramagze V V and Chketiani O G 1988 *Preprint IKI* PR-1440

[5] Nigmatulin R I 1987 *Dynamics in Multiphase Media* vol 1 (Moscow: Nauka) p 2 (in Russian)

[6] Nemtsov B E 1989 *Akustichesky J.* **4**

[7] Krauze F and Rudiger G 1974 *Astron. Nachr.* **295** 93